Intra-Landau-level excitations of the two-dimensional electron–hole liquid

2009 J. Phys.: Condens. Matter 21 235801

(http://iopscience.iop.org/0953-8984/21/23/235801)

IOP PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **21** (2009) 235801 (9pp) doi:10.1088/0953-8984/21/23/235801

Intra-Landau-level excitations of the two-dimensional electron–hole liquid

S A Moskalenko[1], M A Liberman[2], E V Dumanov[1], A G Stefan[1] and M I Shmiglyuk[1]

[1] Institute of Applied Physics of the Academy of Sciences of Moldova, Academic Street 5, Chisinau MD-2028, Republic of Moldova
[2] Department of Physics, Uppsala University, Box 530, SE-751 21 Uppsala, Sweden

Received 16 December 2008, in final form 8 April 2009
Published 11 May 2009
Online at stacks.iop.org/JPhysCM/21/235801

Abstract
The intra-Landau-level excitations of the two-dimensional electron–hole liquid are characterized by two branches of the energy spectrum. The acoustical plasmon branch with in-phase oscillations of electrons and holes has a linear dispersion law in the range of small wavevectors, with a velocity which does not depend on the magnetic field strength, and monotonically increases with saturation at higher values of the wavevectors. The optical plasmon branch with oscillations of electrons and holes in opposite phases has a quadratic dependence in the range of long wavelength, a weak roton-type behaviour at the intermediary values of the wavevectors and monotonically increases with saturation similar to the case of the acoustical branch. The influence of the supplementary in-plane electric field leads to the drift of the charged particles in the crossed electric and magnetic fields and to the energy spectrum as in the reference frame, where the e–h system is moving with the drift velocity. A perturbation theory using the Green function method is developed on the basis of a small parameter $v^2(1-v^2)$, where v^2 is the filling factor and $(1-v^2)$ displays the phase space filling effect.

1. Introduction

The plasma oscillations of the one-component electron gas in the three-dimensional (3D) bulk crystals [1] as well as in the two-dimensional (2D) layers are characterized by the squared frequencies $\omega_{\mathrm{p}}^2(q) = \frac{4\pi e^2 n_{\mathrm{e}}}{\varepsilon_0 m}$ and $\omega_{\mathrm{p}}^2(q) = \frac{2\pi e^2 n_S q}{\varepsilon_0 m}$, where n_{e} and n_S are the corresponding electron densities. Das Sarma and Madhukar [2] have considered the two-component 2D electron gas (2DEG). Two density fluctuation operators $\hat{\rho}_1(q)$ and $\hat{\rho}_2(q)$ correspond to each layer combining in-phase and in opposite phases forming the optical and acoustical plasmon oscillations with the frequencies $\omega_{\mathrm{OP}}(q) \sim \sqrt{q}$ and $\omega_{\mathrm{AP}}(q) \sim q$ in the range of small wavevectors q. Kasyan *et al* [3, 4] have investigated the interconnected plasmon–phonon excitations in 2D structures. The excitations of the 2DEG in the strong perpendicular magnetic field are completely different due to the quenching of the kinetic energy by the magnetic field and Landau quantization.

Collective elementary excitations in the case of a one-component (OC) two-dimensional electron gas including the one-component plasma (OCP) have been investigated in many papers. From the beginning we shall discuss such systems because their energy spectrum is simpler without an excitonic branch, which appears in two-component systems such as the electron–hole system in a single-or double-well structures or in a bilayer electron–electron system; all of them being situated in a strong perpendicular magnetic field. They reveal different types of elementary excitations depending on the properties of their ground states. In the case of 2DEG in a strong perpendicular magnetic field they represent an incompressible quantum liquid (IQL) [5], charge density waves (CDWs) [6], electron lattice or Wigner crystals [7], arrays in the space of Landau orbitals without pinning [8] and others. Girvin, MacDonald and Platzman (GMDP) [9] proposed the magneto-roton theory of collective elementary excitations in 2DEG for the conditions of the fractional quantum Hall effect (FQHE). This state occurs in low-disordered, high-mobility samples with partially filled lowest Landau levels (LLLs) with filling factors of the form $\nu = \frac{1}{q}$, where q is an integer ($q \neq 1$). The excitations are a collective effect arising from the many-body correlations due to Coulomb interaction. Considerable progress has been achieved by Laughlin [5] towards understanding the nature of the many-body ground state due to his variational wavefunctions. The theory of the collective excitation spectrum proposed in [9] is analogous to Feynman's theory of superfluid helium [10]. Feynman's main argument leads to the conclusion that the low-lying excitations of any systems should include density waves. The Feynman–Bijl formula determines the excitation energy $\Delta(k)$, as a ratio of two values $f(k)$ and $s(k)$ in the form $\Delta(k) = \frac{f(k)}{s(k)}$, where k is the wavevector, $f(k)$ is the oscillator strength and $s(k)$ is the static structure factor. Both of them are effects of the correlated motion of the particles and determine such dynamic characteristics of the system as the excitation energy $\Delta(k)$ [9, 10].

Similar dependence of the excitation energy on the properties of the ground states of the systems will be discussed below in other examples. In the case of filled Landau levels $\nu = 1$, the lowest excitations are necessarily the cyclotron modes because of the Pauli exclusion principle, in which particles are excited to the higher Landau levels. This case was studied by Kallin and Halperin [11]. For the case of FQHE and for the fractionally filled LLL, the Pauli principle no longer excludes low-energy intra-Landau-level excitations. The energy spectrum of elementary excitations can be obtained considering the equation of motion for the density fluctuation operators $\hat{\rho}(\vec{Q})$, which means to commute them with the Hamiltonian H of the system. In the case of 2DEG in a strong perpendicular magnetic field the kinetic energy of the electrons is quenched in the framework of the LLL. It could be interpreted as if the derivatives $i\hbar\frac{d\hat{\rho}(\vec{Q})}{dt}$ and the commutator $[\hat{\rho}(Q), \hat{H}]$ are equal to zero. If so, the density operators do not change in time and the density wave oscillations do not exist. This paradox was first discussed by Girvin *et al* [9], who pointed out that in the presence of a strong magnetic field the definition of the density fluctuation operator $\hat{\rho}(\vec{Q})$ is changed and does not coincide with the expression in the absence of the magnetic field. In the paper by Girvin and Jach [12] a general formalism within the LLL in two dimensions was proposed including the new definition of the density fluctuation operators. An alternative definition of the density fluctuation operators proposed by Paquet *et al* [13] will be used in the present paper. The integral density fluctuation operators $\hat{\rho}(\vec{Q})$ in the presence of the magnetic field are composed from the partial operators, each of them with its proper phase different from the phases of other components.

In the case of IQL the values $f(k)$ and $s(k)$ are proportional to k^4 and tend to zero as in the limit $k \to 0$. The dependence $s(k) \sim k^4$ means the incompressibility of the ground state [9]. Side-by-side with IQL other ground states were studied. Levesque *et al* [8] have compared the state of CDW with IQL. The lattice state has lower energy per electron than in the framework of IQL for the filling factor $\nu = \frac{1}{9}$ of the LLL. Tao and Thouless [14] have shown that the correlation energy of a 2DEG in a strong magnetic field may be enhanced if the electrons are regularly arranged in the space of Landau orbitals. A new macroscopic collective state differs from the CDW state. The latter state should be pinned and yields a threshold voltage for electrical conduction. Maki and Zotos [15] have studied the stability of the CDWs at low temperatures. The shear modulus and the phonon spectrum of the electron lattice were calculated. In the long-wavelength limit the lower phonon mode exhibits a dispersion relation of the type $q^{\frac{3}{2}}$ as in the classical lattice in a magnetic field.

The density wave oscillations obtained in the case of 2D electron–hole liquids (EHL) have nothing to do with the exciton branch of the spectrum, having a plasmon-type origin. A review of the papers dedicated to OC2DEG in the absence of a strong perpendicular magnetic field can be found in [16]. The two-component electron–electron and electron–hole 2D systems in a strong perpendicular magnetic field reveal new branches of collective elementary excitations. Fertig [17] has investigated the excitation spectrum of two-layer and three-layer electron systems in a strong perpendicular magnetic field. The case of the two-layer electron–electron system with the total filling factor for both layers equal to 1 was considered. The spontaneous coherence of two-component 2DEG was introduced, which is equivalent to the BCS-type ground state of the superconductor. It represents the coherent pairing of the conduction electrons in one layer with the holes in the same conduction band of another layer. Such unusual excitons are named FQHE excitons. In the asymmetric case, when the distance between the layers is not zero, the energy spectrum of elementary excitations is characterized by linear dependence on the wavevector in the region of long wavelengths as well as by a roton-type behaviour at the intermediate values of the wavevectors. In the symmetric case, when the distance between the layers vanishes and the Coulomb interaction inter-layers and intra-layers are the same, the linear region of the energy spectrum is transformed into the quadratic dependence. Such a type of energy spectrum was also discussed by Joglekar and MacDonald [18]. The 2D electron–hole fluid in a strong perpendicular magnetic field was investigated by Paquet *et al* [13] for the case when the ground state of the system represents the Bose–Einstein condensation of magnetoexcitons on the single-particle state with wavevector $k = 0$. The results obtained earlier by Lerner and Lozovik [19] were explained in [13] using a more simple and transparent derivation. Paquet *et al* have discussed the collective excitations of the condensate described by the BCS-type wavefunction introducing the $\vec{Q}$-dependent density fluctuation operators $\hat{\rho}_e(\vec{Q})$ and $\hat{\rho}_h(\vec{Q})$ for electrons and holes. These authors also introduced the creation and annihilation operators for magnetoexcitons which were used in papers [20, 21]. The equations of motion for these operators in [13] were written using the random phase approximation (RPA), which permits us to linearize them. The RPA and the linearization procedure used in [13] mean that the nonlinear terms are neglected and this does not permit us to take properly into account the noncommutative property of the density operators and its consequences, which have been pointed out first and explored in GMDP [9]. This issue will be discussed in detail in the present paper. In the present paper we will use a more exact equation which is not restricted by linear terms and makes possible to develop the theory of density waves and plasmon oscillations in the 2D e–h system when its ground state is either EHL or BEC of magnetoexcitons.

To better understand the difference between the ground states of a one-component electron gas and of a two-component electron–hole system it is useful to represent on the energy scale the positions of their energies per one particle in dependence on the filling factor $\nu = v^2$. Such behaviour was observed in [22]. In the case of coplanar electrons and holes the photoluminescence (PL) spectrum in the FQHE regime does not exhibit anomalies associated with the FQHE. However, when electron and hole layers are separated a new peak in the PL spectrum is introduced, when the filling factor exceeds a fraction ν_0 at which an IQL occurs. The new peak is separated from the main spectral features by the quasiparticle–quasihole gap. The most important for us are four states such as electron–hole liquid (EHL), Laughlin's incompressible quantum liquid (IQL), the charge density wave (CDW) and the metastable dielectric liquid (MDL) phase [20] formed by the Bose–Einstein condensed magnetoexcitons. The EHL is characterized by the energy per one e–h pair, whereas the MDL is determined by the value of the chemical potential. These two values determine the energy per one e–h pair as a whole and to obtain the energy per one particle it is necessary to divide them by two. Now these values can be compared with the energies per one particle in the OC2DEG, especially in the case of IQL and CDW as they were determined by Laughlin in [5] for the fractional filling factors $\frac{1}{3}$ and $\frac{1}{5}$. These values are represented in figure 1. One can observe that at the points $\nu = \frac{1}{3}$ and $\frac{1}{5}$ the IQL has lower energies per one particle than the CDW. The state of the EHL at the points $\nu = \frac{1}{3}$ and $\frac{1}{5}$ is less stable than the previous two one-component states, but has a lowering dependence on the filling factor of the energy per one particle, which reaches the value $-\frac{1}{2}\sqrt{\frac{\pi}{2}}\frac{e^2}{\varepsilon_0 l}$ at the point $\nu = v^2 = 1$. In the interval $\frac{1}{2} < \nu = v^2 < 1$, the existence of the EHL state is of special interest. In the range $0 < \nu = v^2 < \frac{1}{2}$ the MDL is of special interest in the case of the two-component e–h system, though it requires a special description and will not be discussed in the present paper. The four states can be transformed into each other by changing two parameters d and $\nu = v^2$.

2. The Hamiltonian and equations of motion for the operators. Green's functions

The Hamiltonian of the Coulomb interaction of the electrons and holes in the frame of the lowest Landau levels has the form [20]

$$H = \frac{1}{2}\sum_{\vec{Q}} W_{\vec{Q}}[\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q}) - \hat{N}_e - \hat{N}_h]. \quad (1)$$

Here $W_{\vec{Q}}$ is the Fourier transform of the Coulomb interaction, and $\hat{N}_e$ and $\hat{N}_h$ are operators of the full numbers of electrons and holes. The density fluctuation operators for electrons $\hat{\rho}_e(\vec{Q})$ and for holes $\hat{\rho}_h(\vec{Q})$ as well as their linear combinations $\hat{\rho}(\vec{Q})$ and $\hat{D}(\vec{Q})$ are determined as follows:

$$\begin{aligned}
\hat{\rho}_e(\vec{Q}) &= \sum_t e^{iQ_y t l^2} a^\dagger_{t-\frac{Q_x}{2}} a_{t+\frac{Q_x}{2}};\\
\hat{\rho}_h(\vec{Q}) &= \sum_t e^{iQ_y t l^2} b^\dagger_{t+\frac{Q_x}{2}} b_{t-\frac{Q_x}{2}};\\
\hat{\rho}(\vec{Q}) &= \hat{\rho}_e(\vec{Q}) - \hat{\rho}_h(-\vec{Q});\\
\hat{D}(\vec{Q}) &= \hat{\rho}_e(\vec{Q}) + \hat{\rho}_h(-\vec{Q}).
\end{aligned} \quad (2)$$

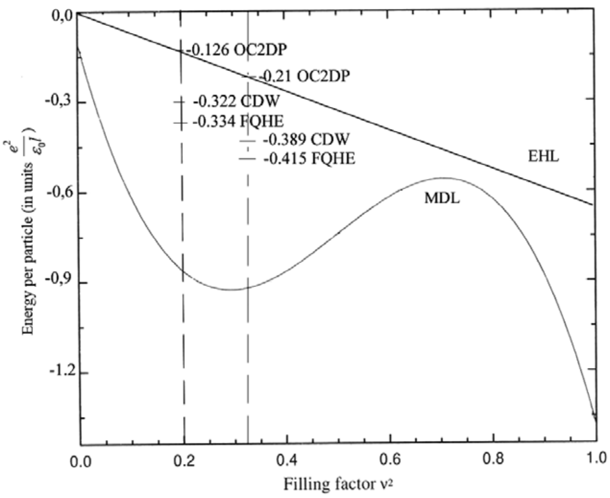

Figure 1. Energies per one particle in units ($\frac{e^2}{\varepsilon_0 l}$) in four different ground states of the OC2DEG and the 2D e–h system such as: incompressible quantum liquid (IQL) arising in the condition of FQHE, charge density wave (CDW), one-component two-dimensional plasma (OC2DP) with the properties of the electron–hole liquid (EHL) and the metastable dielectric liquid (MDL) phase formed by Bose–Einstein condensed 2D magnetoexcitons.

They are expressed through the Fermi creation and annihilation operators $a^\dagger_p, a_p$ for electrons and $b^\dagger_p, b_p$ for holes. The e–h operators depend on two quantum numbers. Only the electrons and holes on the lowest Landau levels $n_e = n_h = 0$ are considered and their notations are dropped. The quantum number p denotes the N-fold degeneracy of the Landau levels in the Landau gauge. $N = \frac{S}{2\pi l^2}$, where S is the surface layer area and l is the magnetic length $l^2 = \frac{\hbar c}{eH}$.

The density fluctuation operators (2) with different wavevectors $\vec{P}$ and $\vec{Q}$ do not commute, which is related to the helicity or spirality accompanying the presence of the strong magnetic field [18]. They are expressed by the phase factors in the structure of operators (2) and by the vector product of two 2D wavevectors $\vec{P}$ and $\vec{Q}$ and its projection in the direction of the magnetic field. These properties considerably influence the structure of the equations of motion for the operators and determine new aspects of the 2D electron–hole (e–h) physics. The equations of motion for the operators $\hat{\rho}(\vec{P})$ and $\hat{D}(\vec{P})$ can be written in different ways with different free terms. One possibility is

$$\begin{aligned}
i\hbar\frac{d\hat{\rho}(\vec{P})}{dt} &= [\hat{\rho}(\vec{P}), H] = -i\sum_{\vec{Q}} W_{\vec{Q}} \sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\\
&\quad\times [\hat{\rho}(\vec{Q})\hat{\rho}(\vec{P}-\vec{Q}) + \hat{\rho}(\vec{P}-\vec{Q})\hat{\rho}(\vec{Q})];\\
i\hbar\frac{d\hat{D}(\vec{P})}{dt} &= [\hat{D}(\vec{P}), H] = -i\sum_{\vec{Q}} W_{\vec{Q}} \sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\\
&\quad\times [\hat{\rho}(\vec{Q})\hat{D}(\vec{P}-\vec{Q}) + \hat{D}(\vec{P}-\vec{Q})\hat{\rho}(\vec{Q})].
\end{aligned} \quad (3)$$

Another form of the equations of motion containing free energy terms outside the nonlinear components is

$$\begin{aligned} i\hbar\frac{d\hat{\rho}(\vec{P})}{dt} &= E(\vec{P})\hat{\rho}(\vec{P}) - 2i\sum_{\vec{Q}} W_{\vec{Q}} \\ &\quad\times \sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\hat{\rho}(\vec{Q})\hat{\rho}(\vec{P}-\vec{Q}); \\ i\hbar\frac{d\hat{D}(\vec{P})}{dt} &= E(\vec{P})\hat{D}(\vec{P}) - 2i\sum_{\vec{Q}} W_{\vec{Q}} \\ &\quad\times \sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\hat{\rho}(\vec{Q})\hat{D}(\vec{P}-\vec{Q}). \end{aligned} \tag{4}$$

Here $E(\vec{P})$ is a quantum of the Coulomb electron–electron interaction in the presence of a strong magnetic field equal to

$$E(\vec{P}) = 2\sum_{\vec{Q}} W_{\vec{Q}} \sin^2\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right). \tag{5}$$

As long as the solution of the equations of motion is exact, the particular choice of how the equations of motion are written, either (3) or (4), is not important. However, because any solution is approximate the initial forms of the equations of motion are important and their physical meanings must be understood from the very beginning. Some comparison of this situation with another physical process could be useful and will be discussed below.

The separation of the free energy part $E(\vec{P})$ in equations (4) is similar to the rearrangement of the Hamiltonians in the mean-field approximation, when forming different quadratic parts with different meanings of the selected quasiparticles. Thereby in the case of electron–electron and electron–phonon interactions we can take into account first of all the electron–electron interaction if it is stronger than the electron–phonon interaction, or we can introduce the polaron picture and then consider the interaction between the polarons. It is useful to compare the 2D electron–hole (e–h) system with the one-component 2D electron gas (2DEG) in the conditions of the fractional quantum Hall effect (FQHE), as well as with the electron system in the field of laser radiation, or with the Bose gas in the condition of Bose–Einstein condensation (BEC). For all these three examples we deal with the system in the presence of unlimited reservoirs of energy which give rise to new quasi-energy branches of the energy spectrum [23] and composite particles [24, 25]. For instance, in the case of FQHE the total flux of the magnetic field equals N flux quanta $\phi_0 = \frac{2\pi\hbar c}{e}$. Each flux quanta creates a vortex in the 2DEG. The associations of the electrons and some flux quanta give rise to charged composite bosons (CB) or composite fermions (CF) [24]. The flux quanta also persist in the case of the 2D e–h system. For example, in the case of one half-filling factor $\nu = \frac{1}{2}$ there are two flux quanta per each e–h pair or per each magnetoexciton. Neglecting the Coulomb interaction between the electron and the hole one can represent a neutral composite e–h pair as being formed by composite charged particles in two variants. In one of them we have a composite fermion formed by the electron and by two flux quanta and a simple hole or vice versa. Another variant is the equal distribution of the flux quanta between the electron and the hole leading to the formation of two charged bosons, when the electron is associated with one flux quantum and the hole with another one. These two variants could coexist simultaneously. Such a composite e–h pair is a quaternion consisting of two quasiparticles and two flux quanta. It is similar with the biexciton, which is also a quaternion, but formed by two electrons and two holes. The similarity between them can be expected by introducing the Coulomb interaction. As was shown in [26] the biexciton formation can be viewed as a pairing of two excitons or of one charged particle with the opposite sign trions. Returning to the composite e–h pair one can compare the charged composite boson with the exciton and the composite charged fermion with the trion. We can expect new interesting properties in these conditions.

Our investigations shown that the plasmon-type quasi-energy complexes with free energy $E(P)$ have large damping rates and do not exist in reality. The only variant without the damping in the framework of the method used is the variant of equations (3) without the free energy $E(P)$. This result can be understood if one takes into account that $E(P)$ does not depend on the filling factor ν^2 of the lowest Landau levels (LLLs), and cannot be regarded as the energy of the intra-Landau-level plasmon-type excitations even in the zero-order approximation of the perturbation theory.

As was pointed out, see, for example, [27], perturbation theory based on the electron–electron Coulomb interaction in a strong magnetic field does not exist. One cannot introduce a small parameter $\frac{V_c}{T_k}$ expressed by the ratio of the potential energy V_c to the kinetic energy T_k, because the kinetic energy is quenched, ($T_k = 0$), by the strong magnetic field in the framework of the LLLs. As usual in the Green's function method, the free energy $\varepsilon(P)$ determines the zero-order Green's functions $G^0(P,\omega) = \frac{1}{\hbar\omega-\varepsilon(P)+i\delta}$. The development of perturbation theory is based on the choice of the small parameter $\nu^2(1-\nu^2)$ as well as of the zero-order Green's functions. They are completely different in both cases (3) and (4). All these considerations justify our choice in the favour of the variant (3). To the best of our knowledge these aspects have not been discussed in the literature before.

We will introduce the retarded Green's functions at temperature $T = 0$ as

$$\begin{aligned} G_1(\vec{P},t) &= \langle\langle\hat{\rho}(\vec{P},t);\, \hat{X}^\dagger(\vec{P},0)\rangle\rangle; \\ G_2(\vec{P},t) &= \langle\langle\hat{D}(\vec{P},t);\, \hat{X}^\dagger(\vec{P},0)\rangle\rangle. \end{aligned} \tag{6}$$

They consist of two time-dependent operators $\hat{\rho}(\vec{P},t)$ and $\hat{D}(\vec{P},t)$, which can be named as active participants of the Green's function structure, and of one idle term, which is denoted as $\hat{X}^\dagger(\vec{P},0)$. The former one does not depend on time, being taken at the point $t = 0$. The Green's functions are determined by the relation

$$\begin{aligned} &G(t) = \langle\langle\hat{A}(t);\, \hat{B}(0)\rangle\rangle = -i\theta(t)\langle[\hat{A}(t);\, \hat{B}(0)]\rangle; \\ &\hat{A}(t) = e^{\frac{iHt}{\hbar}}\hat{A}e^{-\frac{iHt}{\hbar}}; \quad [\hat{A}(t);\, \hat{B}(0)] = \hat{A}\hat{B} - \hat{B}\hat{A}; \end{aligned} \tag{7}$$

where $\hat{H}$ is the Hamiltonian (1). The average $\langle\,\rangle$ will be calculated using the ground state wavefunction of the electron–hole liquid (EHL), which means to substitute the average occupation numbers of electrons and holes on the LLLs by the filling factor v^2.

3. Self-energy parts and the dispersion laws

Using Zubarev's procedure for the Green's function, which is describing in the appendix, we obtain the closed Dyson equation for the Green's function $G_1(\vec{P}, \omega)$:

$$G_1(\vec{P}, \omega)\Sigma_{11}(\vec{P}, \omega) = C \tag{8}$$

where the self-energy part $\Sigma_{11}(\vec{P}, \omega)$ has the form

$$\Sigma_{11}(\vec{P}, \omega) = (\hbar\omega + \mathrm{i}\delta) - \frac{4}{\hbar\omega + \mathrm{i}\delta}\sum_{\vec{Q}} W_{\vec{Q}}(W_{\vec{Q}} - W_{\vec{P}-\vec{Q}}) \times \sin^2\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle. \tag{9}$$

The average $\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle$ is calculated when the ground state represents the EHL at the temperature $T = 0$. This state is characterized by the average occupation numbers for electrons and holes $\langle a_p^\dagger a_p\rangle = \langle b_p^\dagger b_p\rangle = v^2$ and leads to the values

$$\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle = 2Nv^2(1 - v^2); \quad \langle\hat{\rho}(\vec{Q})\hat{D}(-\vec{Q})\rangle = 0. \tag{10}$$

Taking into account the vanishing value of the mixed average $\langle\hat{\rho}\hat{D}\rangle = 0$ we obtain the closed equation for the second Green's function:

$$G_2(\vec{P}, \omega)\Sigma_{22}(\vec{P}, \omega) = C \tag{11}$$

where $\Sigma_{22}(\vec{P}, \omega)$ equals

$$\Sigma_{22}(\vec{P}, \omega) = (\hbar\omega + \mathrm{i}\delta) - \frac{4}{\hbar\omega + \mathrm{i}\delta} \times \sum_{\vec{Q}} W_{\vec{Q}}^2 \sin^2\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle. \tag{12}$$

As one can see both the self-energy parts have only infinitesimal imaginary parts iδ, which means that both the elementary excitations are without damping in a given approximation. The variant with the free energy term $E(P)$ would lead to a damping of the same order as the value of the real part.

The second Green's function $G_2(\vec{P}, \omega)$ describes the intra-LLL excitations of acoustical type where the electron and hole density fluctuations take place in phase. Their energy spectrum is characterized by the dispersion law

$$(\hbar\omega_{\mathrm{AP}}(\vec{P}))^2 = 4\sum_{\vec{Q}} W_{\vec{Q}}^2 \sin^2\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right) \times \langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle. \tag{13}$$

Now we will express in dimensionless forms the wavevector $Pl = x$ and excitation energy $\frac{\hbar\omega_{\mathrm{AP}}(x)}{I_l}$, where l is the magnetic length, $I_l = \sqrt{\frac{\pi}{2}}e^2/\varepsilon_0 l$ is the ionization potential of the magnetoexciton and ε_0 is the dielectric constant of the medium. Instead of (13) we will write [29]

$$\left(\frac{\hbar\omega_{\mathrm{AP}}(x)}{I_l}\right)^2 = 2v^2(1 - v^2)V_1(x);$$
$$V_1(x) = \frac{1}{2\pi}\left[G + \Gamma\left(0, \frac{x^2}{4}\right) + \mathrm{Ln}\left(\frac{x^2}{4}\right)\right];$$
$$\Gamma(0, x) = -E_i(-x);$$
$$E_i(x) = G + \mathrm{Ln}(-x) + \sum_{k=0}^{\infty}\frac{x^k}{kk!}. \tag{14}$$

Here $G = 0.577\,216$ is the Euler constant and $\Gamma(0, x)$ is the incomplete Gamma function.

In the range of small values of $x < 1$, $V_1(x) = \frac{x^2}{8\pi}$ and the acoustical plasmon branch has a linear dispersion. In the range of large values of $x > 1$, $V_1(x)$ is monotonically increasing with the saturation function. It is worth mentioning that the velocity of the acoustical plasmon branch c_0 equals

$$c_0 = \frac{e^2}{2\hbar\epsilon_0}\sqrt{\frac{v^2(1 - v^2)}{2}}. \tag{15}$$

It does not depend on the magnetic field strength, but only on the filling factor. In the case $\epsilon_0 = 13$, $v^2 = \frac{1}{3}$, c_0 equals 6×10^6 cm s^{-1}. It can be compared with the drift velocity V_{d} discussed in section 4.

Another branch of the excitation spectrum corresponds to optical plasmons, where the electron and hole density fluctuations take part in opposite phases.

The optical plasmon branch has a dispersion law

$$(\hbar\omega_{\mathrm{OP}}(\vec{P}))^2 = 4\sum_{\vec{Q}} W_{\vec{Q}}(W_{\vec{Q}} - W_{\vec{P}-\vec{Q}}) \times \sin^2\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle. \tag{16}$$

Taking into account that the most probable value of Q obeys the condition $\overline{Ql} \simeq 1$, we have expanded the expression under the integral in the range $x = Pl < \overline{Ql} = 1$ and in the range $x \geqslant 1$. In these two regions of x variation we have obtained

$$\frac{(\frac{\hbar\omega_{\mathrm{OP}}(x)}{I_l})^2}{v^2(1 - v^2)} = \begin{cases} 2x^2V_1(x), & x < 1 \\ 2V_1(x) - \left[\tilde{E}(x) - U_1(x) \times \left(1 + \frac{1}{x^2}\right)\right]\sqrt{\frac{2}{\pi}}(\mathrm{e}^{-\frac{x^2}{2}})\Big/ x, & x > 1 \end{cases} \tag{17}$$

where

$$\tilde{E}(x) = \frac{E(x)}{I_l} = \left(1 - \mathrm{e}^{-\frac{x^2}{4}}I_0\left(\frac{x^2}{4}\right)\right);$$
$$U_1(x) = \tfrac{1}{4}\mathrm{e}^{-\frac{x^2}{4}}\left(2\mathrm{e}^{\frac{x^2}{4}} + (x^2 - 2) \times I_0\left(\frac{x^2}{4}\right) - x^2 I_1\left(\frac{x^2}{4}\right)\right). \tag{18}$$

Here $I_i(x)$ are the modified Bessel functions.

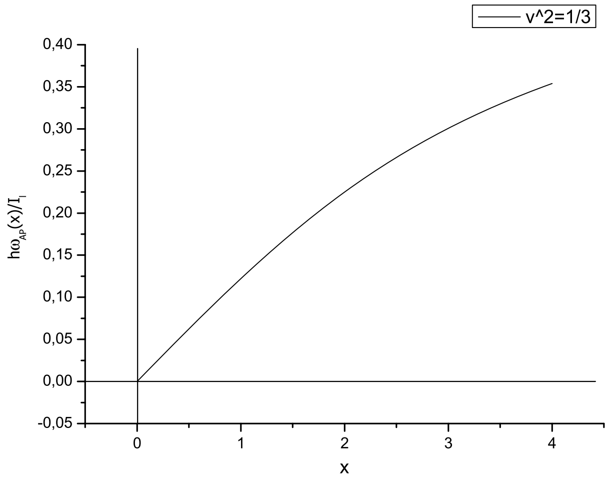

Figure 2. The dispersion law for the acoustical plasmon branch.

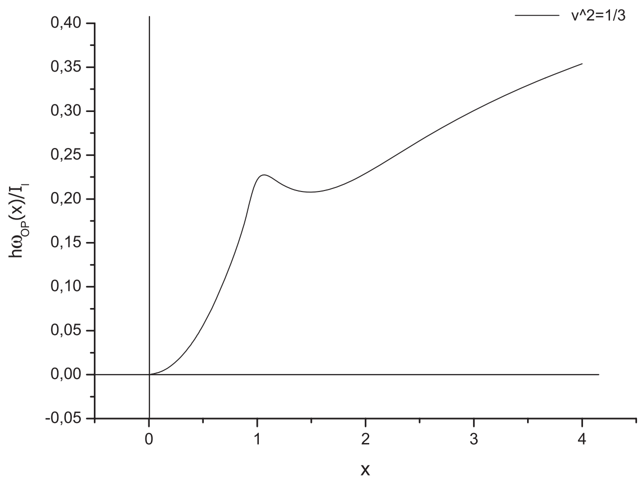

Figure 3. The dispersion law for the optical plasmon branch.

Taking into account that in the range $x < 1$, $V_1(x) = \frac{x^2}{8\pi}$ we find $2x^2V_1(x) = \frac{x^4}{4\pi}$ and a quadratic dependence on x of the optical plasmon frequency. The dispersion laws for acoustical and optical plasmon-type intra-LLL excitations are shown in figures 2 and 3.

The optical branch has a quadratic dependence on x in the range $x < 1$, a nonmonotonic behaviour of the faint roton type in the intermediary region and a monotonic increase with saturation at greater values of the variable x. Let us compare our findings with the results of the papers [13, 17, 18]. The main difference concerns the proportionality of the plasmon frequencies to the parameter $v^2(1-v^2)$. Though the plasmon branches do not depend on the excitonic-type energy spectrum, nevertheless their dependences on the wavevectors can be compared. In [13, 18] the bilayer electron system was described as a double quantum well with spatially separated electrons and holes in the same conduction band with the filling factor $\nu = v^2 = \frac{1}{2}$ for each component. In the asymmetric case the electrons and holes are separated in different layers, which diminishes the inter-layer electron–hole interaction in comparison with the intra-layer interactions of the homogeneous particles. In this asymmetric case the repulsion prevails over the attraction and the dispersion law as the energy spectrum in [17] becomes linear in the region of small wavevectors. With vanishing distance between the layers, the linear dispersion in [17] transforms into the quadratic dispersion law.

In our case the electrons in the conduction band and the holes in the valence band are situated on the same layer and the electron–electron, hole–hole and electron–hole Coulomb interactions are exactly the same in absolute value as in the symmetric case considered in [17]. This explains the origin of the quadratic dispersion law for the optical plasmon in the region of long wavelengths obtained in our paper. The acoustical plasmon branch has different contributions of the Hartree and Fock Coulomb interaction terms in comparison with the optical plasmon branch and its dispersion law in the region of long wavelengths is linear.

4. 2D electron–hole system in a strong perpendicular magnetic field and a lateral electric field

We consider now the energy spectrum when a lateral electric field is added supplementary to the strong perpendicular magnetic field. Let the in-plane electric field be oriented in the direction of Landau quantization in the Landau gauge. In this case the wavefunctions of the electrons and holes in a crossed magnetic and electric field in the Landau gauge are [30]

$$\psi^i_{p,n}(y) = \frac{\mathrm{e}^{\mathrm{i}px}}{\sqrt{L_x}}\varphi^i_{n,p}(y);$$
$$\varphi^i_{n=0,p}(y) = \frac{1}{\sqrt{l\sqrt{\pi}}}\exp\left[-\frac{(y-y^i_p)^2}{2l^2}\right]; \tag{19}$$

where

$$y^i_p = \frac{q_i}{e}l^2\left(-p + \frac{m_iV_\mathrm{d}}{\hbar}\right); \qquad i = e, h; \quad q_i = \mp e. \tag{20}$$

The energy spectrum of the electrons and holes, which move with the drift velocity V_d, is the following:

$$E^i_{n,p} = -\frac{m_iV_\mathrm{d}^2}{2} + \hbar V_\mathrm{d}p + \hbar\omega_{ci}\left(n+\frac{1}{2}\right); \qquad V_\mathrm{d} = c\frac{E}{H}. \tag{21}$$

It depends linearly on the one-dimensional continuous wavevector p directed perpendicular to both crossed fields, in the same direction as the drift velocity. The density fluctuation operators are denoted as $\rho^\mathrm{E}_i(\vec{Q})$ and are determined by the previous expressions multiplied by the phase factors

$$\hat{\rho}^\mathrm{E}_i(\vec{Q}) = \mathrm{e}^{-\mathrm{i}Q_yu_i}\hat{\rho}_i(\vec{Q}); \qquad u_i = \frac{V_\mathrm{d}}{\omega_{ci}}; \quad \omega_{ci} = \frac{eH}{m_ic}. \tag{22}$$

Their linear combinations are introduced as

$$\hat{\rho}^\mathrm{E}(\vec{Q}) = \hat{\rho}^\mathrm{E}_\mathrm{e}(\vec{Q}) - \hat{\rho}^\mathrm{E}_\mathrm{h}(-\vec{Q});$$
$$\hat{D}^\mathrm{E}(\vec{Q}) = \hat{\rho}^\mathrm{E}_\mathrm{e}(\vec{Q}) + \hat{\rho}^\mathrm{E}_\mathrm{h}(-\vec{Q}). \tag{23}$$

The Hamiltonian describing 2D electrons and holes in crossed magnetic and electric fields has the form

$$H^{\mathrm{E}} = H_0 + H^{\mathrm{E}}_{\mathrm{Coul}};$$
$$H_0 = \sum_p \hbar V_{\mathrm{d}} p (a^\dagger_p a_p + b^\dagger_p b_p); \tag{24}$$
$$H^{\mathrm{E}}_{\mathrm{Coul}} = \frac{1}{2}\sum_{\vec{Q}} W_{\vec{Q}} [\hat{\rho}^{\mathrm{E}}(\vec{Q})\hat{\rho}^{\mathrm{E}}(-\vec{Q}) - \hat{N}_{\mathrm{e}} - \hat{N}_{\mathrm{h}}].$$

In this case the energy spectrum of the plasma excitations can be obtained substituting the frequency $\hbar\omega(P)$ by the difference $\hbar\omega(P) - \hbar V_{\mathrm{d}} P_x$, as in the reference frame, where the e–h system is moving with the drift velocity V_{d}.

The drift velocity V_{d} is less than the sound velocity c_0 if

$$E < \frac{c_0}{c} H \approx 2 \times 10^{-4} H.$$

For example, for $H = 10$ T, this means $E < 6000$ V. The energy spectrum of the acoustical plasmon branch is stable if the drift velocity V_{d} does not exceed c_0. The optical plasmon energy spectrum is unstable in the region of small wavevectors, where $\hbar\omega_{\mathrm{OP}}(P) < \hbar V_{\mathrm{d}} P$. This means that the system will emit spontaneously optical plasmons on account of the external electric field, trying to diminish its drift velocity.

5. Conclusions

The plasmon-type intra-lowest-Landau-level (LLL) excitations of the 2D EHL formed on the surface of the layer and subjected to the action of a strong perpendicular magnetic field are characterized by two branches of the energy spectrum. Their frequencies are proportional to the generalized filling factor $v^2(1 - v^2)$ reflecting the filling of the LLLs and the phase space filling effect. These collective elementary excitations have different origins and different values compared to the excitonic-type collective elementary excitations in the case of the BEC of 2D magnetoexcitons. In the case of EHL the acoustical plasmon branch has a linear dispersion law in the range of small wavevectors and it monotonically increases with saturation at higher wavevectors. The second branch of the elementary excitations is an optical plasmon branch with quadratic dispersion law at small wavevectors with faint roton-type dispersion at intermediary wavevectors and with a similar behaviour as the acoustical branch at higher wavevectors. When a lateral electric field is supplementarily applied to the system, the dispersion laws of the energy spectrum can be obtained by substituting the energy $\hbar\omega(P)$ by $\hbar\omega(P) - \hbar V_{\mathrm{d}} P_x$ for both branches. Such an energy spectrum is usual in the reference frame where the e–h system is moving with the drift velocity V_{d}. The energy spectrum of the acoustical plasmon branch is stable when the drift velocity V_{d} does not exceed c_0. The optical plasmon energy spectrum is unstable in the region of the small wavevectors, where $\hbar\omega_{\mathrm{OP}}(P) < \hbar V_{\mathrm{d}} P$. It means that the system will emit spontaneously optical plasmons on account of the external electric field, trying to diminish its drift velocity.

Acknowledgments

This research was supported by the joint grant of the Russian Foundation for Fundamental Research and of the Academy of Sciences of Moldova (08.820.05.033RF). One of the authors (SAM) greatly appreciates the support of the Wenner–Gren Foundation during his stay in Uppsala, where this work has been completed. SAM is grateful to Professors P I Khadzhi and A I Kasyan for the valuable suggestions and useful discussions. EVD thanks the Foundation for Young Scientists of the Academy of Sciences of Moldova for financial support (08.819.05.07F).

Appendix

The Fourier transforms of the Green's functions (6) are denoted as

$$G_1(\vec{P}, t) = \langle\langle \hat{\rho}(\vec{P}) | \hat{X}^\dagger(\vec{P}) \rangle\rangle_\omega;$$
$$G_2(\vec{P}, t) = \langle\langle \hat{D}(\vec{P}) | \hat{X}^\dagger(\vec{P}) \rangle\rangle_\omega. \tag{A.1}$$

The equations of motion for the Green's functions (A.1) are

$$(\hbar\omega + \mathrm{i}\delta) G_1(\vec{P}, \omega) = C - \mathrm{i}\sum_{\vec{Q}} W_{\vec{Q}} \sin\left(\frac{[\vec{P} \times \vec{Q}]_z l^2}{2}\right)$$
$$\times \langle\langle [\hat{\rho}(\vec{P} - \vec{Q})\hat{\rho}(\vec{Q}) + \hat{\rho}(\vec{Q})\hat{\rho}(\vec{P} - \vec{Q})] | \hat{X}^\dagger(\vec{P}) \rangle\rangle_\omega$$
$$(\hbar\omega + \mathrm{i}\delta) G_2(\vec{P}, \omega) = C - i\sum_{\vec{Q}} W_{\vec{Q}} \sin\left(\frac{[\vec{P} \times \vec{Q}]_z l^2}{2}\right) \tag{A.2}$$
$$\times \langle\langle [\hat{D}(\vec{P} - \vec{Q})\hat{\rho}(\vec{Q}) + \hat{\rho}(\vec{Q})\hat{D}(\vec{P} - \vec{Q})] | \hat{X}^\dagger(\vec{P}) \rangle\rangle_\omega.$$

Here, all average values which do not contain the Green's functions were denoted by C. Now we can formulate the unambiguous requirement for the Green's functions determined by the Dyson equations (8) and (11) through the self-energy parts $\Sigma_{ii}(P, \omega)$. In the zeroth-order approximation the general expression must be in agreement with the properties of the electron and hole operators whose dependences on time are $a^{(0)}_p(t) = \mathrm{e}^{-\frac{1}{2}\omega_{\mathrm{ce}} t} a_p(0)$; $b^{(0)}_p(t) = \mathrm{e}^{-\frac{1}{2}\omega_{\mathrm{ch}} t} b_p(0)$, which lead to the lack of time dependence of the density fluctuation operators $\rho^{(0)}(P, t) = \rho(P, 0)$; $D^{(0)}(P, t) = D(P, 0)$ and to $G^{(0)}_{1,2}(P, \omega) = \frac{1}{\hbar\omega + \mathrm{i}\delta}$. As one can see only the equation of motion (3) is compatible with this requirement. The starting one-operator Green's functions (A.1) contain only one operator on the left-hand side as regards the vertical line.

The equations of motion (A.2) lead to the appearance of two-operator Green's functions of the types $\langle\langle \hat{\rho}\hat{\rho} | \hat{X} \rangle\rangle_\omega$ and $\langle\langle \hat{D}\hat{\rho} | \hat{X} \rangle\rangle_\omega$, for which the new equations of motion will be derived. This procedure is well known in Zubarev's variant of the Green's function method [28] and leads to the formation of an infinite chain of equations of motion, which require to be truncated. This will be done below, expressing for example, the three-operator Green's function approximated by the one-operator Green's function multiplied by the products of the average values of the remaining two operators, which contain the small parameter of the perturbation theory $\nu^2(1 - \nu^2)$. In this way we obtain the exact equation of motion for the two-operator Green's function:

$$\langle\langle\hat\rho(\vec P-\vec Q)\hat\rho(\vec Q)+\hat\rho(\vec Q)\hat\rho(\vec P-\vec Q)|\hat X^\dagger(\vec P)\rangle\rangle_\omega(\hbar\omega+\mathrm{i}\delta)$$
$$=C-\mathrm{i}\sum_{\vec R}W_{\vec R}\sin\left(\frac{[(\vec P-\vec Q)\times\vec R]_z l^2}{2}\right)$$
$$\times\langle\langle\hat\rho(\vec P-\vec Q-\vec R)\hat\rho(\vec R)\hat\rho(\vec Q)+\hat\rho(\vec R)\hat\rho(\vec P-\vec Q-\vec R)$$
$$\times\hat\rho(\vec Q)+\hat\rho(\vec Q)\hat\rho(\vec P-\vec Q-\vec R)\hat\rho(\vec R)$$
$$+\hat\rho(\vec Q)\hat\rho(\vec R)\hat\rho(\vec P-\vec Q-\vec R)$$
$$\times|\hat X^\dagger(\vec P)\rangle\rangle_\omega-\mathrm{i}\sum_{\vec R}W_{\vec R}\sin\left(\frac{[\vec Q\times\vec R]_z l^2}{2}\right)$$
$$\times\langle\langle\hat\rho(\vec P-\vec Q)\hat\rho(\vec R)\hat\rho(\vec Q-\vec R)$$
$$+\hat\rho(\vec R)\hat\rho(\vec Q-\vec R)\hat\rho(\vec P-\vec Q)$$
$$+\hat\rho(\vec Q-\vec R)\hat\rho(\vec R)\hat\rho(\vec P-\vec Q)$$
$$+\hat\rho(\vec P-\vec Q)\hat\rho(\vec Q-\vec R)\hat\rho(\vec R)|\hat X^\dagger(\vec P)\rangle\rangle_\omega. \qquad (A.3)$$

Substituting this expression into the first equation (A.2) we obtain

$$-\mathrm{i}\sum_{\vec Q}W_{\vec Q}\sin\left(\frac{[\vec P\times\vec Q]_z l^2}{2}\right)\langle\langle\hat\rho(\vec P-\vec Q)\hat\rho(\vec Q)$$
$$+\hat\rho(\vec Q)\hat\rho(\vec P-\vec Q)|\hat X^\dagger(\vec P)\rangle\rangle_\omega$$
$$=C-\frac{1}{\hbar\omega+\mathrm{i}\delta}\sum_{\vec Q}\sum_{\vec R}W_{\vec Q}W_{\vec R}\sin\left(\frac{[\vec P\times\vec Q]_z l^2}{2}\right)$$
$$\times\sin\left(\frac{[(\vec P-\vec Q)\times\vec R]_z l^2}{2}\right)\langle\langle\hat\rho(\vec P-\vec Q-\vec R)\hat\rho(\vec R)\hat\rho(\vec Q)$$
$$+\hat\rho(\vec R)\hat\rho(\vec P-\vec Q-\vec R)\hat\rho(\vec Q)+\hat\rho(\vec Q)\hat\rho(\vec P-\vec Q-\vec R)\hat\rho(\vec R)$$
$$+\hat\rho(\vec Q)\hat\rho(\vec R)\hat\rho(\vec P-\vec Q-\vec R)|\hat X^\dagger(\vec P)\rangle\rangle_\omega$$
$$-\frac{1}{\hbar\omega+\mathrm{i}\delta}\sum_{\vec Q}\sum_{\vec R}W_{\vec Q}W_{\vec R}\sin\left(\frac{[\vec P\times\vec Q]_z l^2}{2}\right)$$
$$\times\sin\left(\frac{[\vec Q\times\vec R]_z l^2}{2}\right)\langle\langle\hat\rho(\vec P-\vec Q)\hat\rho(\vec R)\hat\rho(\vec Q-\vec R)$$
$$+\hat\rho(\vec R)\hat\rho(\vec Q-\vec R)\hat\rho(\vec P-\vec Q)$$
$$+\hat\rho(\vec Q-\vec R)\hat\rho(\vec R)\hat\rho(\vec P-\vec Q)+\hat\rho(\vec P-\vec Q)$$
$$\times\hat\rho(\vec Q-\vec R)\hat\rho(\vec R)|\hat X^\dagger(\vec P)\rangle\rangle_\omega. \qquad (A.4)$$

A similar expression was derived for the two-operator Green's functions entering the second equation of motion (A.2). The truncations and the decoupling of the three-operator Green's functions were made using the approximations

$$\langle\langle\hat\rho(\vec P-\vec Q-\vec R)\hat\rho(\vec R)\hat\rho(\vec Q)|\hat X^\dagger(\vec P)\rangle\rangle_\omega$$
$$\approx\langle\langle\hat\rho(\vec P)|\hat X^\dagger(\vec P)\rangle\rangle_\omega[\delta_{kr}(\vec Q,\vec P)\langle\hat\rho(\vec R)\hat\rho(-\vec R)\rangle$$
$$+(\delta_{kr}(\vec R,-\vec Q)+\delta_{kr}(\vec R,\vec P))\langle\hat\rho(\vec Q)\hat\rho(-\vec Q)\rangle];$$
$$\langle\langle\hat\rho(\vec P-\vec Q)\hat\rho(\vec R)\hat\rho(\vec Q-\vec R)|\hat X^\dagger(\vec P)\rangle\rangle_\omega$$
$$\approx\langle\langle\hat\rho(\vec P)|\hat X^\dagger(\vec P)\rangle\rangle_\omega[\delta_{kr}(\vec Q,0)\langle\hat\rho(\vec R)\hat\rho(-\vec R)\rangle$$
$$+(\delta_{kr}(\vec R,\vec Q-\vec P)+\delta_{kr}(\vec R,\vec P))\langle\hat\rho(\vec P-\vec Q)\hat\rho(\vec Q-\vec P)\rangle];$$
$$\langle\langle\hat\rho(\vec R)\hat\rho(\vec Q-\vec R)\hat D(\vec P-\vec Q)|\hat X^\dagger(\vec P)\rangle\rangle_\omega$$
$$\approx\langle\langle\hat D(\vec P)|\hat X^\dagger(\vec P)\rangle\rangle_\omega\delta_{kr}(\vec Q,0)\langle\hat\rho(\vec R)\hat\rho(-\vec R)\rangle$$
$$+\langle\langle\hat\rho(\vec P)|\hat X^\dagger(\vec P)\rangle\rangle_\omega[\delta_{kr}(\vec R,\vec P)$$
$$+\delta_{kr}(\vec R,\vec Q-\vec P)]\langle\hat\rho(\vec Q-\vec P)\hat D(\vec P-\vec Q)\rangle;$$
$$\langle\langle\hat\rho(\vec R)\hat\rho(\vec Q)\hat D(\vec P-\vec Q-\vec R)|\hat X^\dagger(\vec P)\rangle\rangle_\omega$$
$$\approx\langle\langle\hat D(\vec P)|\hat X^\dagger(\vec P)\rangle\rangle_\omega\delta_{kr}(\vec R,-\vec Q)\langle\hat\rho(\vec Q)\hat\rho(-\vec Q)\rangle$$
$$+\langle\langle\hat\rho(\vec P)|\hat X^\dagger(\vec P)\rangle\rangle_\omega[\delta_{kr}(\vec R,\vec P)\langle\hat\rho(\vec Q)\hat D(-\vec Q)\rangle$$
$$+\delta_{kr}(\vec Q,\vec P)\langle\hat\rho(\vec R)\hat D(-\vec R)\rangle]. \qquad (A.5)$$

They permit us to represent the nonlinear terms in the right-hand sides of equations (A.2) in the following ways:

$$-\mathrm{i}\sum_{\vec Q}W_{\vec Q}\sin\left(\frac{[\vec P\times\vec Q]_z l^2}{2}\right)\langle\langle\hat\rho(\vec P-\vec Q)\hat\rho(\vec Q)$$
$$+\hat\rho(\vec Q)\hat\rho(\vec P-\vec Q)|\hat X^\dagger(\vec P)\rangle\rangle_\omega\approx C+\frac{4}{\hbar\omega+\mathrm{i}\delta}$$
$$\times\sum_{\vec Q}W_{\vec Q}(W_{\vec Q}-W_{\vec P-\vec Q})\sin^2\left(\frac{[\vec P\times\vec Q]_z l^2}{2}\right)$$
$$\times\langle\hat\rho(\vec Q)\hat\rho(-\vec Q)\rangle G_1(\vec P,\omega);$$
$$-\mathrm{i}\sum_{\vec Q}W_{\vec Q}\sin\left(\frac{[\vec P\times\vec Q]_z l^2}{2}\right)\langle\langle\hat D(\vec P-\vec Q)\hat\rho(\vec Q)$$
$$+\hat\rho(\vec Q)\hat D(\vec P-\vec Q)|\hat X^\dagger(\vec P)\rangle\rangle_\omega$$
$$\approx C+\frac{4}{\hbar\omega+\mathrm{i}\delta}\sum_{\vec Q}\sum_{\vec R}W^2_{\vec Q}\sin^2\left(\frac{[\vec P\times\vec Q]_z l^2}{2}\right)$$
$$\times\langle\hat\rho(\vec Q)\hat\rho(-\vec Q)\rangle G_2(\vec P,\omega). \qquad (A.6)$$

On their basis the final expressions (9) and (12) for the self-energy parts $\Sigma_{11}(\vec P,\omega)$ and $\Sigma_{22}(\vec P,\omega)$ were obtained.

References

[1] Pines D 1963 *Elementary Excitations in Solids* (New York: Benjamin)
[2] Das Sarma S and Madhukar A 1981 *Phys. Rev.* B **23** 805
[3] Kasyan A I and Rusu P I 1985 *Phys. Status Solidi* b **127** 439
[4] Kasyan A I, Sur I V and Balmush I N 1991 *Semiconductors* **25** 689 (in Russian)
[5] Laughlin R B 1983 *Phys. Rev. Lett.* **50** 1395
[6] Yoshioka D and Lee P A 1983 *Phys. Rev.* B **27** 4986
[7] Wigner E P 1934 *Phys. Rev.* **46** 1002
[8] Levesque D, Weis J J and MacDonald A H 1984 *Phys. Rev.* B **30** 1056
[9] Girvin S M, MacDonald A H and Platzman P M 1986 *Phys. Rev.* B **33** 2481
[10] Feynman R P 1972 *Statistical Mechanics* (Reading, MA: Benjamin) chapter 11
[11] Kallin C and Halperin B I 1984 *Phys. Rev.* B **30** 5655
[12] Girvin S M and Jach T 1984 *Phys. Rev.* B **29** 5617
[13] Paquet D, Rice T M and Ueda K 1985 *Phys. Rev.* B **32** 5208
Rice T M, Paquet D and Ueda K 1985 *Helv. Phys. Acta* **58** 410
[14] Tao R and Thouless D J 1983 *Phys. Rev.* B **28** 1142
[15] Maki K and Zotos X 1983 *Phys. Rev.* B **28** 4349
[16] Baus M and Hansen J-P 1980 *Phys. Rep.* **59** 1–94
[17] Fertig H A 1989 *Phys. Rev.* B **40** 1087
[18] Joglekar Y N and MacDonald A H 2001 *Phys. Rev.* B **64** 155315
[19] Lerner V and Lozovik Yu E 1981 *Zh. Eksp. Teor. Fiz.* **80** 1488
Lerner V and Lozovik Yu E 1981 *Sov. Phys.—JETP* **53** 763 (Engl. Transl.)
[20] Moskalenko S A, Liberman M A, Snoke D W and Botan V 2002 *Phys. Rev.* B **66** 245316
[21] Moskalenko S A, Liberman M A, Khadzhi P I, Dumanov E V, Podlesny Ig V and Botan V 2007 *Physica* E **39** 137–149

[22] MacDonald A H, Rezayi E A and Keller D 1992 *Phys. Rev. Lett.* **68** 1939
[23] Baz' A I, Zeldovich Ya B and Perelomov A M 1971 *Scattering, Reactions and Decays in Nonrelativistic Quantum Mechanics* (Moscow: Nauka) (in Russian)
[24] Jain J K 1989 *Phys. Rev. Lett.* **63** 199
[25] Stormer H L 1999 *Rev. Mod. Phys.* **71** 875–89
[26] Moskalenko S A and Snoke D W 2000 *Bose–Einstein Condensation of Excitons and Biexcitons and Coherent Nonlinear Optics with Excitons* (Cambridge: Cambridge University Press) pp 1–415
[27] Apalkov V M and Rashba E I 1992 *Phys. Rev.* B **46** 1628
Apalkov V M and Rashba E I 1993 *Phys. Rev.* **48** 18312
Apalkov V M and Rashba E I 1991 *Pis. Zh. Eksp. Teor. Fiz.* **54** 160
Apalkov V M and Rashba E I 1992 *Pis. Zh. Eksp. Teor. Fiz.* **55** 38
[28] Zubarev D N 1960 *Usp. Fiz. Nauk* **71** 71 (in Russian)
[29] Gradshteyn I S and Ryzhik I M 1965 *Tables of Integrals, Sums, Series and Products* (New York: Academic)
[30] Giuliani G F and Vignali G 2005 *Quantum Theory of the Electron Liquid* (Cambridge: Cambridge University Press) chapter 10